SHAHEED BENAZIR BHUTTO WOMEN UNIVERSITY Peshawar

DEPARTMENT OF MATHEMATICS

DETAILED COURSE OUTLINE OF MATHEMATICS M.SC (2 YEARS PROGRAM)2017 AND ONWARDS

Course Name: Real Analysis-I	**Course Code:** MTH-514
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites: Calculus-III	

Specific Objectives of course:

This is the first course in analysis. It develops the fundamental ideas of analysis and is aimed at developing the students' ability in reading and writing mathematical proofs. Another objective is to provide sound understanding of the axiomatic foundations of the real number system, in particular the notions of completeness and compactness.

Course Outline:

Number Systems: Ordered fields. Rational, real and complex numbers. Archimedean property, supremum, infimum and completeness.

Topology of real numbers: Convergence, completeness, completion of real numbers. Open sets, closed sets, compact sets. Heine Borel Theorem. Connected sets.

Sequences and Series of Real Numbers: Limits of sequences, algebra of limits. Bolzano Weierstrass Theorem. Cauchy sequences, liminf, limsup. Limits of series, convergences tests, absolute and conditional convergence. Power series.

Continuity: Functions, continuity and compactness, existence of minimizers and maximizers, uniform continuity. Continuity and connectedness, Intermediate mean Value Theorem. Monotone functions and discontinuities.

Differentiation: Mean Value Theorem, L'Hopital's Rule, Taylor's Theorem.

Recommended Books:

- S. Lang, Analysis *I*, Addison-Wesley Publ. Co., Reading, Massachusetts, 1968.
- W. Rudin, *Principles of Mathematical Analysis*, 3rd ed., Mc.Graw Hill, 1976.
- B. S. Thomson, J. B. Bruckner and A. M. Bruckner, *ElementaryReal Analysis*, 2nd Ed. 2008.
- G. Boros, V. Moll, *Irresistible Integrals: Symbolics, Analysisan Experiments in the Evaluation of Integrals*, Cambridge University Press, 2004.
- J. Borwein, D. Bailey, R. Girgenson, *Experimentation inMathematics: Computational Paths to discovery*, Wellesley, MA, A.K. Peters, 2004.
- G. Bartle , R. Sherbert , *Introduction to Real Analysis*, 3rd edition, John Wiley, New York, 1999.
- Widder, D.v. (1982). *Advanced Calculus:* Prentice-Hall

Course Name: Topology-I	**Course Code:** MTH-522
Course Structure: Lecture: 3	**Credit Hours: 03**
Prerequisites:	

Course Outline

Topological Spaces
History, Construction of topology, Open sets, Closed sets, Limit points, Neighborhoods, Subspace topology, Interior and Exterior points,
Metric Spaces:
Generalization of the concept of distance, Metric of real line, plane and spaces, Open, Closed structure in Metric spaces, Sequences in Metric spaces, Cauchy sequences,
Continuity and Homeomorphism:
Generalization of Continuity, Continuity in topological spaces, Continuity in Metric spaces, Restriction function, Homeomorphism between spaces, Composition of functions,
Basis and Sub basis:
Generation of Topology through Basis, Sub basis, Generation of topology through sub basis.

Recommended Books:

- VV Prasolov,(1995). *Intutive Topology:* American Mathematical Society.
- J.R.Weeks,(2002*). The Shape of Space:* 2nd ed., Marcel Derkker.
- J.Dugundji, (1996). *Topology*: Boston:Allyn and Bacon
- Willet and Mankir's.

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Course Name: Group Theory-I	**Course Code:** MTH-521
Course Structure: Lectures: 3	**Credit Hours:** 3

Course Outline:

This course includes the fundamental concepts of groups, properties of groups, Centralizer and normalizer, Subgroups, Cosets, Lagrange's Theorem, Cyclic group, Homomorphism, Isomorphism Theorems, Permutation theory, Permutations and Caley's Theorem.

Recommended Books:

- Willard Miller, (1972). *Symmetry of Groups and their Applications*: Academic press.
- Joseph J. Rotman, (1994). *An Introduction to Group Theory*: 4th ed., Springer.

Course Name: Complex Analysis-I	**Course Code:** MTH-524
Course Structure: Lectures: 3	**Credit Hours:** 3

Prerequisites:

Course Outline:

This course includes the fundamental concepts of Complex Numbers, limit points and closure of a set Interior, exterior and boundary of a set, Complex Functions Limits in complex plane, Continuity in complex plane, Uniform continuity, Derivatives, Analytic functions, Singular points, Cauchy Riemann equations, Cauchy Riemann equation in polar form, Elementary functions, logarithmic and exponential functions, line integral and Cauchy integral formula and its application.

Recommended Book

- Churchill, R.V.Verhey and Brown R., (1996). *Complex Variable and Applications:* Mc.Graw Hill
- Marsden, J.E, (1982). *Basic Complex Analysis:* W.H Freeman and Co

Course Name: Real Analysis-II	**Course Code:** MTH-515
Course Structure: Lectures: 3	**Credit Hours:** 3

Prerequisites: Real Analysis-I

Specific Objectives of the course:

A continuation of Real Analysis-I, this course will continue to cover the fundamentals of real analysis, concentrating on the Riemann Stieltjes integrals, Functions of Bounded Variation, Improper Integrals, and convergence of series. Emphasis would be on proofs of main results.

Course Outline:

Riemann Integrals and Riemann Integrals as limit of sum.
Explicit and implicit functions.
Limits of functions of several variables: Continuity of functions of partial derivatives of higher order.
Jacobians.
Extreme values: Maxima and Minima.

Recommended Books:

- S. Lang, *Analysis I, II*, Addison-Wesley Publ. Co., Reading,Massachusetts,1968,1969.
- W. Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw-Hill, 1976.
- K. R. Davidson and A. P. Donsig, *Real Analysis withReal Applications*, Prentice Hall Inc., Upper Saddle River, 2002.
- G. B. Folland, *Real Analysis*, 2nd Edition, John Wiley and Sons, New York, 1999.
- E. Hewitt and K. Stromberg, *Real and Abstract Analysis*, Springer-Verlag, Berlin Heidelberg New York, 1965.
- H. L. Royden, *Real Analysis*, 3rd Edition, Macmillan, New York, 1988.
- G. Bartle , R. Sherbert , *Introduction to Real Analysis*, 3rd edition, John Wiley, New York, 1999.

Course Name: Topology-II	**Course Code:** MTH-523
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites:	

Course Outline:

Compact spaces:
Generalization of the concept of finiteness and closedness, Compact Spaces, Open covering, Finite sub covers, Continuity in compact space, Compact subspaces, Locally compact spaces, Heine Borel theorem,

Connected Spaces:
Separated Sets, Connected and Disconnected spaces, Components, Locally connected spaces, Path Connected spaces,

Separation Axioms:
Separation Axioms,T0 , T1 spaces, Hausdorff Space, Normal spaces, Completely normal spaces, Regular spaces, Completely regular spaces, Compactness in Hausdorff spaces, Continuity, Topological Invariant properties, Topologically preserved properties, Hereditary properties.

Complete Metric Spaces:

Cauchy sequence, Convergent Cauchy sequence, Complete Metric spaces, Space of continuous functions, Space of bounded sequences, L^p space.

- **Recommended Books:**
  - VV Prasolov,(1995). *Intutive Topology :* American Mathematical Society.
  - J.R.Weeks,(2002*). The Shape of Space:* 2nd ed., Marcel Derkker.
  - J.Dugundji, (1996). *Topology*: Boston:Allyn and Bacon

Course Name: Linear Algebra	**Course Code:** MTH-526
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites:	

Course Outline:

This course includes vector spaces, basis and dimension, linear dependence and linear independence, linear transformation, Invertible and singular elements, algebra of linear transformation and eigenvalues and eigen vectors, Minimal polynomial, and matrices of linear transformation.

Recommended Books:

- H.anton, and C.Rorres, (2011). *Elementary Linear Algebra*:10th ed., Wiley.
- H.Anton, (2005). *Elementary Linear Algebra*: 9thed.,Wiley.

Course Name: Complex Analysis- II	**Course Code:** MTH-525
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites:	

Course Outline:

This course includes the fundamental concepts of Power Series with its types, Poles and Residues, Improper Integrals with its types, Gemma, Beta and Hyper Geometric and Legendre Functions.

Recommended Books:

- Churchill, R.V.Verhey and Brown R., (1996). *Complex Variable and Applications:* Mc.Graw Hill
- Marsden, J.E, (1982). *Basic Complex Analysis:* W.H Freeman and Co

Course Name: Numeric and Symbolic Computation	**Course Code:** MTH-527
Course Structure: Lectures: 1, Practical: 2	**Credit Hours: (**1+2)
Prerequisites:	

Course Outline:

Introduction

General introduction and basic use of mathematica, numeric and symbolic computation, the note book, working with data, input and output, built-in functions, front end and the kernel, errors, help

Language of Mathematica

Expressions, values, variables, functions and assignments, immediate vs delayed, patterns and pattern matching, conditional patterns, predicates and Boolean operations, relational and logical operators, attributes.

Lists

Simple and multidirectional list, List construction and manipulation, testing a list, extracting elements, rearranging list, list component assignments, working with several lists

Programming

Functional programming, Map, Thread, Apply, Inner and Outer, Nest, NestList,

Programs as functions, user-defined functions, pure functions, module. Procedural programming, loops, flow control. Rule base programming. Dynamic programming. Graphics programming. Writing packages.

Recommended Books

- Paul R. Wellin, Richard J. Gaylord, Samuel N. Kamin, *An introduction to programming with Mathematica, third edition*, Cambridge university press New York, 2005.
- Hartmut F. W. Hoft, Margret Hoft, *Computing with Mathematica, second edition*. Academic Press, 2003.
- Martha L. Abell, James P.Braselton, *Mathematica By Example, Third Edition,* Academic Press, 2004.

Course Name: Numerical Analysis-I	**Course Code:** MTH- 611
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites:	

Course Outline:

Operators:

Introduction to Error, Forward operator, Backward operator, Shift operator, Average operator, Relations between operators, Difference tables, Factorial Polynomials.

Interpolation (Equally Spaced):

Newton forward formula, Stirling's formula, Newton Backward formula, Gauss's interpolation formula,

Interpolation (Unequally Spaced):

Newton divided difference table, Newton formula, Lagrange Interpolating formula, Cubic splines, Inverse interpolation.

Numerical Differentiation:

Differentiation formulas based on Newton, Stirling's formulas, Differentiation based on Newton divided formulas, Higher order differentiation formulas, Richardson's Extrapolation formulas, Error analysis of higher order formulas.

Numerical Integration:

Newton Quadrature formulas, Trapezoidal rule, Simpson's rule, Error analysis, Romberg Integration.

Recommended Books:

- James B. Scarborough(1986). *Numerical Analysis*: The John Hopkins Press.
- C.F. Gerald, (1984). *Applied Numerical Analysis*: Addision

Wesley.

- R.L. Burden, J.D. Fires and A.C. Reynolds, (1981). *Numerical Analysis*: Boston: Prindle, Weber and Schmidt.

Course Name: (Rings and fields)	**Course Code:** MTH-619
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites:	
Course Outline: This course includes the Isomorphism Theorems, Conjugacy classes, Characteristic subgroups, Direct Product of Groups, Fully invariant subgroups, Groups of Automorphisms and Direct product of groups	
Recommended Books: • Alperin, J.L., and Bell, R.B., (1995).*Group and Representation*: New York: Springer-Verlag. • Dixon, J.D., (1973). *Problems in Group theory*: New York: Dover. • Robinson, D.J.S, (1995). *A Course in theory of Groups*: 2nd ed.,New York: Springer-Verlag.	

Course Name: Functional Analysis-I	**Course Code:** MTH-621
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites:	
Course Outline: This course includes Normed Linear Spaces, Inequalities, Bounded linear operators ,Topologically isomorphism,Equivalent norm, Theorems on finite dimensional normed linear spaces, F.Reisz's lemma, Banach space, Dual Space, Quotient space, principle of uniform boundedness, Hahn-Banach theorem for non linear spaces, Inner product space, Open Mapping theorem, Closed graph theorem, Schwartz's inequality, Polarization identity, Parallelogram law and orthogonal sets.	
Recommended Books: • R. Beattie,(2007). *Convergence Structure and Applications to Functional Analysis:* Kulwer academic Publishers. • Erwin Kreyzig, (1989). *Introductory Functional Analysis*: Wiley and Sons. • Semen Samsonovich Kutateladze,(1996). *Fundamentals of Functional Analysis:* Kulwer Academic Publishers	

Course Name: Measure Theory –I	**Course Code:** MTH-623
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites: Real Analysis	

Course Outline:

This course includes the Fundamental concepts of measurable functions and sets. Denumerable Sets, Cardinal numbers, Measurable sets and their theorems, Measurable function, Extended definition of measurable function, Function of bounded variation, Absolutely Continuous

Recommended Books:

- Doob.J.L. (1994). *Measure Theory*: New York: Springer- Verlag
- Evans, L.C and Gariepy, R.F. (1992). *Measure Theory and Fine Properties of Functions*: Boca Raton, FL: CRC Press.

Course Name: Ordinary Differential Equation	**Course Code:** MTH-614
Course Structure: Lectures: 3	**Credit Hours: 0**4
Prerequisites: Calculus-I	

Specific Objectives of the course:

To introduce students to the formulation, classification of differential equations and existence and uniqueness of solutions. To provide skill in solving initial value and boundary value problems. To develop understanding and skill in solving first and second order linear homogeneous and nonhomogeneous differential equations and solving differential equations using power series methods.

Course Outline:

Preliminaries: Introduction and formulation, classification of differential equations, existence and uniqueness of solutions, introduction of initial value and boundary value problems

First order ordinary differential equations: Basic concepts, formation and solution of differential equations. Separable variables, Exact Equations, Homogeneous Equations, Linear equations, integrating factors. Some nonlinear first order equations with a known solution, differential equations of Bernoulli and Ricaati type, Clairaut equation, modeling with first-order ODEs, Basic theory of systems of first-order linear equations, Homogeneous linear system with constant coefficients, Nonhomogeneous linear system

Second and higher order linear differential equations: Initial value and boundary value problems, Homogeneous and non-homogeneous equations, Superposition principle, homogeneous equations with constant coefficients, Linear independence and Wronskian, Nonhomogeneous equations, undetermined coefficients method, variation of parameters, Cauchy-Euler equation, Modeling.

Laplace transform: Introduction and properties of Laplace transform, transforms of elementary functions, periodic functions, error function and Dirac delta function, inverse Laplace transform, convolution theorem, solution of PDEs by Laplace transform, Diffusion and wave equations

Series Solutions: Power series, ordinary and singular points, Existence of power series solutions, power series solutions, types of singular points, Frobenius theorem, Existence of Frobenius series solutions, solutions about singular points, The Bessel, modified Bessel, Legendre and Hermite equations and their solutions.

Recommended Books:

- Dennis G. Zill and Michael R., Differential equations with boundary-value problems by Cullin 5th Edition Brooks/Cole, 1997.
- William E. Boyce and Richard C. Diprima, Elementary differential equations and boundary value problems, Seventh Edition John Wiley & Sons, Inc
- V. I. Arnold, *Ordinary Differential Equations*, Springer, 1991.
- T. Apostol, *Multi-Variable Calculus and Linear Algebra*, 2nd ed., John Wiley and sons, 1997.

Course Name: Numerical Analysis –II	**Course Code:** MTH-612
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites:	

Course Outline:

Numerical solution of Linear Equations:

Introduction to Direct and Iterative methods, Jacobi method, Gauss seidel method, SOR method, Rate of convergence.

Numerical solution of Nonlinear Equations:

Bisection method, Newton raphson method, Secant method, Regula Falsi method, Rate of Convergence and order of convergence.

Numerical Solution to ODE's (Initial Value)

Single Step Methods
Euler method, Taylor's methods, R-K 2 method, Heun's Method, R-K 4 method, Truncation Errors.
Muti Step Methods
Adam Bashforth method, Adam Moulton method, Truncation Errors.

Numerical Solution to ODE's (Initial Boundary Value)
Ray Leigh Ritz method, Finite difference method.
Eigen Values:

Numerically finding Eigen values, Properties and applications, Power method, Jacobi method,

Recommended Books:

- James B. Scarborough(1986). *Numerical Analysis*: The John Hopkins Press.
- C.F. Gerald, (1984). *Applied Numerical Analysis*: Addision Wesley.
- R.L. Burden, J.D. Fires and A.C. Reynolds, (1981). *Numerical Analysis*: Boston: Prindle, Weber and Schmidt.

Course Name: Rings and fields	**Course Code:** MTH-619
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites:	

Course Outline:

This course includes the syllow theorems, simple groups, normal and composition series, solvable groups, Jorden holder theorem lower, Refinement theorems and upper central series and Ring Theory.

Recommended Books

- Alperin, J.L., and Bell, R.B., (1995).*Group and Representation*: New York: Springer-Verlag.
- Dixon, J. D., (1973). *Problems in Group theory*: New York: Dover.
- Robinson, D.J.S, (1995). *A Course in theory of Groups*: 2nd ed., New York: Springer-Verlag.

Course Name: Functional Analysis-II	**Course Code:** MTH-622
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites: Analysis and Topology	

Course Outline:

This course includes Hilbert Spaces, Orthogonality, Orthonormal sets, Bessel's inequality, Partially ordered set, Gram-Smidt orthogonalization process, Operators on Hilbert Spaces, projection, finite dimensional spectral theory

Recommended Books:

- R. Beattie,(2007). *Convergence Structure and Applications to Functional Analysis:* Kulwer Academic Publishers.
- Erwin Kreyzig, (1989). *Introductory Functional Analysis*: Wiley and Sons.
- Semen Samsonovich Kutateladze,(1996). *Fundamentals of Functional Analysis:* Kulwer Academic Publishers

Course Name: Measure Theory –II	**Course Code:** MTH-624
Course Structure: Lectures: 3	**Credit Hours:** 3
Prerequisites: Real Analysis	

Course Outline:

This course includes Lebesgue integral, Lebesgue integral for bounded functions, convergence theorems, the fundamental theorem of integral calculus, the Lebesgue integral for unbounded functions, Derivatives, Nondifferentiable functions of bounded variations of absolute continuous functions, indefinite integrals mean convergence and the lebesgue classics L^p.

Recommended Books:

- Doob.J.L. (1994). *Measure Theory*: New York : Springer- Verlag
- Evans, L.C and Gariepy, R.F. (1992). *Measure Theory and Fine Properties of Functions*: Boca Raton, FL: CRC Press.

Course Name: Partial Differential Equations	**Course Code:**MTH-615
Course Structure: Lectures: 3	**Credit Hours:** 4
Prerequisites: Ordinary Differential Equations	

Specific Objectives of the course:

Partial Differential Equations (PDEs) are at the heart of applied mathematics and many other scientific disciplines. The course aims at developing understanding about fundamental concepts of PDEs theory, identification and classification of their different types, how they arise in applications, and analytical methods for solving them. Special emphasis would be on wave, heat and Laplace equations.

Course Outline:

First order PDEs: Introduction, formation of PDEs, solutions of PDEs of first order, The Cauchy's problem for quasilinear first order PDEs, First order nonlinear equations, Special types of first order equations

Second order PDEs: Basic concepts and definitions, Mathematical problems, Linear operators, Superposition, Mathematical models: The classical equations, the vibrating string, the vibrating membrane, conduction of heat solids, canonical forms and variable, PDEs of second order in two independent variables with constant and variable coefficients, Cauchy's problem for second order PDEs in two independent variables

Methods of separation of variables: Solutions of elliptic, parabolic and hyperbolic PDEs in Cartesian and cylindrical coordinates

Fourier transforms: Fourier integral representation, Fourier sine and cosine representation, Fourier transform pair, transform of elementary functions and Dirac delta function, finite Fourier transforms, solutions of heat, wave and Laplace equations by Fourier transforms.

Recommended Books:

- Myint UT, *Partial Differential Equations for Scientists and Engineers*, 3rd edition, North Holland, Amsterdam, 1987.

- Dennis G. Zill, Michael R. Cullen, *Differential equations with boundary value problems,* Brooks Cole, 2008.
- John Polking, Al Boggess, *Differential Equations with Boundary Value Problems*, 2nd Edition, Pearson, July 28, 2005.
- J. Wloka, *Partial Differential Equations*, Cambridge University press, 1987.